### Measuring the Mass of the Top Quark: Answers

By 1974 four quarks had been detected. They were; **up** (0.0024 GeV), **down** (0.0048 GeV), **strange** (0.104 GeV) and **charm** (0.127 GeV). The charm quark was discovered simultaneously in 1974 by two rival American teams - Brookhaven on the east coast and SLAC on the west coast. After the **bottom** quark (4.2 GeV) was discovered in 1977 at Fermilab, teams around the world raced to see who would be the first to detect its partner – the **top** quark.

1. Based on the above information, when would you expect the top quark to be discovered and approximately what would its mass be?

Physicists expected that the top quark would be detected in just a few years with a mass similar to the bottom quark. The students should come to a similar conclusion. This is nothing like what happened.

At Fermilab protons and anti-protons were accelerated to identical high speeds and smashed together. They annihilated into pure energy and sometimes this energy would produce a top-antitop pair.

TheseTheseThese quarks decayed almost immediately into a few large particles, which then decayed almost immediately into jets of particles. The momenta of the particles are shown on the data sheet. There are five sources of momenta detected – a muon and four jets of particles.

1. The original proton and anti-proton were moving into the page with equal speeds from opposite sides. In what possible directions could the top and anti-top quarks they formed be moving?
2. any direction b) any opposite direction c) any opposite direction in the plane of the paper

Explain:

Momentum must be conserved. The particles could be produced in any direction as long as they moved opposite each other. The example we will examine, conveniently happened to have almost all the momentum in one plane.

1. When you add the six momenta you should find that they add to

a) around 300 GeV b) around 0 GeV c) exactly 0 GeV d) a different number each time

Explain:

Answer a) has made the error of adding the momenta as scalars. Answer d) might be a result of a student thinking about the random nature of collisions or quantum events.

Answer c) comes from conservation of momentum. If the momentum was zero before the collision, then it should be zero afterwards.

Answer b) recognizes conservation of momentum but is also aware that this is a real experiment with experimental errors.

Students will probably wonder about the units. At this stage, you can just point out that it doesn’t matter what units you use, as long as you are consistent. A more detailed explanation is given before question five.

1. Draw vectors representing each of the six momenta in the correct direction using a scale of 1.0 cm: 10 GeV. Measure the horizontal and vertical components of each and enter them in the table below. Enter the total momentum in each direction in the final column.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | 95.5 | 54.8 | 17.0 | 61.2 | 65.9 | Total | Total  (GeV/c) |
| Horizontal  (mm) | -93 | -33 | 11 | 58 | 21 | -36 | -36 |
| Vertical  (mm) | -14 | 44 | 14 | 20 | 62 | 2 | 2 |

These have been colour-coded to match the vectors drawn on the data sheet.

1. The detectors were designed to detect all known particles except neutrinos. What is the momentum of the missing neutrino? Draw this momentum on the data sheet.

It will be to the right and slightly down in order to make the momentum of the final column add to zero. It has a magnitude of around (362 + 022)1/2= 36 GeV. Each student will get a slightly different value. It might be worth discussing why. The biggest source of error in the analysis is probably from estimating the direction.

1. Relativity says that mass, energy and momentum are related by the following equation E2 = (pc)2 + (mc2)2. When the top and antitop quarks decayed they produced particles that were moving really fast and therefore their masses were negligible compared to their momenta. For example, the mass of a muon is 0.106 GeV, but the muons in this experiment have momenta of 7.3 and 61.2 GeV.
2. Set the mass to zero and simplify the equation. E = pc
3. Find the total energy of all of the particles including the neutrino. 330 GeV
4. The collision of the proton and antiproton only had just enough energy to produce the mass of top and anti-top quarks. There was no spare energy for momentum.
5. Set the momentum to zero and simplify the equation. E = mc2

b) What is the mass of a top quark?

The total energy of 330 GeV has to be divided by two because a top and an anti-top were produced. The top mass from my calculation was 165 GeV.

1. In 1995 the top quark’s mass was found to be 172.0 +/- 2.2 GeV/c2.   
   a) How do the date and mass compare to what you predicted in the first question? (Note: Physicists were also surprised.)

Physicists expected that the top quark would be detected in just a few years with a mass similar to the bottom quark. The students should come to a similar conclusion. This is nothing like what happened. For 13 years, CERN in Switzerland and Fermilab in the US, were in direct competition, producing collisions of ever higher energies in the search for the top quark. In 1990 CERN had to cede the race to Fermilab because they had reached the energy limits of the LEP (Large Electron Positron collider) and wanted to shut it down to reconfigure it as the LHC (Large Hadron Collider). At this point it was known that the mass of the top quark mass must be greater than 77 GeV – over 15 times more massive than the bottom quark. The top quark was finally discovered in 1995 at Fermilab with a mass that is similar to a tungsten atom!

1. Calculate your percentage error in the top quark’s mass.

A mass of 165 GeV gives an error of - 4%. Not bad!

c) What was the major source of error?

The error should be negative because this was a 3-D collision. This particular collision was almost but not completely 2-D. Therefore some of the momentum was out of the page and was missed by our calculations. The physicists at Fermilab had to do this calculation in 3-D.

1. In order to find the top quark, Fermilab collided a proton and an antiproton together at very high speeds. You have a friend who studies biology. Your friend thinks that this is a rather sloppy way to dissect protons to see what is inside them. Explain to your friend how this collision is not like a dissection. Be sure to refer to the masses of protons and top quarks.

The top quark is not a particle that is found inside a proton. The top quarks mass is 180 times greater than a proton’s mass of 0.938 GeV! When the fast moving particles collide, all of their energy - from mass and motion - is turned into pure energy. The collision was just the way that physicists got a lot of energy packed into a tiny space.

1. Lancaster University has a simulation that lets you look at data to find evidence for a Higgs particle. Your job is to find collisions with these two photons, measure their energies and use the energies and the angle between them to determine the mass of the particle that formed them. There are a lot of other interactions taking place, so there is a lot of noise in the data. You need to look at many examples for your results to be statistically significant.
2. Go to <http://www.lppp.lancs.ac.uk/higgs/higgs.html> Go to Measurement and scroll to the bottom of the page. Press fire. You should see an image similar to the top quark data. You are supposed to find two green towers, without any lines leading to them – that is the signature of two photons. Can you find them? Probably not. Go to options and select an energy cutoff of 20 GeV and then elect fire. Try other energy cut-offs. What energy cut-off lets you find photon events most rapidly? What happens when it is too high or too low?

Setting a high energy cutoff simplifies the diagram greatly by removing the lower energy towers. However, if it is set too high it may cut off the photons that you are trying to detect. A value of around 50 GeV seems to be pretty good.

1. Find an event with two photon towers. Now you need to calculate the mass of the particle that created the two photons. Select *measure energies* and click on the two photon towers. They should turn yellow. Then select *measure angle* and click on the angle. Finally, select *calculate mass*. The mass will then be displayed on a histogram on the bottom left. To confirm the detection of a Higgs particle you need to get a lot of events. Your challenge is to collect as many events in 10 minutes as possible and to develop the largest significance value. How many did you find? Click on the fit button in the bottom right if it is red. What was the significance of your findings? You are aiming for a significance greater than 5.

I just tried it. I got 29 events, with a significance of 1.6. I used an energy cutoff of 50 GeV. Then I selected *auto events* and rapidly got 71 events with a significance of 2.8. It told me that 57 of the 99 generated events were discarded because the photon energies were below the cutoff. I think that I would avoid having the students use the auto event until they had done 10 minutes of hand collection. You get a better sense of how the events vary and how much work is involved and I expect my students will get very competitive about this. The ‘auto events’ often doesn’t work.