**Accelerating Frames of Reference and Time**

1. Watch The GPS and Relativity [perimeterinstitute.ca/en/Perimeter\_Inspirations/General/Perimeter\_Inspirations/](http://perimeterinstitute.ca/en/Perimeter_Inspirations/General/Perimeter_Inspirations/)
2. How is time on the GPS satellite affected by special relativity (speed)?

Key Point: The satellite is moving so we see its time slowed.

Side Issue: According to special relativity, shouldn’t they see our time slowed as well? This is true for inertial frames. However, the satellite is accelerating and breaks the symmetry. This is an example of the twin paradox and the twin that accelerates, is the one that has less time pass.

1. How is time on the GPS satellite affected by general relativity (gravity)?

Key Point: The satellite is farther from the Earth and so the effects of gravity are less than for us.

Side Issue: The satellite’s time is slowed less so, their clock seems to run faster. The gravitational speeding up is greater than the relativistic slowing down. It is possible for them to cancel. If it was closer to the Earth, the gravitational speeding up would be less. Closer to the Earth, the satellite would move faster and so the relativistic slowing down would increase. So why didn’t they put the satellite at this magic radius? There is not much advantage to doing this – setting the clocks to always run slowly is easy. Plus, the orbit is chosen so it orbits over the same point twice each day. This allows them check them twice a day without having to aim.

1. The effects of relativity on time are really small and usually don’t matter unless you go close to the speed of light. Give two reasons why they matter for the GPS?

We are interested in distances, not time. Any error in time is multiplied by the speed of light – a very large number. The errors keep accumulating so that after a day the distance is off by 11 km.

To understand why gravity affects time, we need to consider accelerating frames of reference.

1. A plastic water bottle has a small hole in the top. It is turned upside down and dropped with the hole uncovered. What happens to the water as the bottle falls?

a) It pours out at the same rate. b) It pours out slower.

c) It pours out faster. d) It stays in the bottle.

Explain:

Most students after a chance to think and argue about this realize that the water stays in the bottle.

1. The bottle is thrown upwards. What happens to the water while the bottle is in the air?

a) It pours out on both the way up and down. b) It stays in the bottle except when in your hands

c) It pours out only on the way up. d) It pours out only on the way down.

Explain:

This question is much harder for students. They get velocities (up and down) confused with acceleration (only down). They may have had experiences in falling down – off a high diving board perhaps – but not falling up and down. Freefall is not just for falling down. Freefall is whenever only gravity is acting on an object. Projectile motion is also free fall. The students could demonstrate this with the bottle.

1. A cup of water is on a tray. The tray is swung rapidly in a horizontal circle. The water stays in the cup and the cup stays on the tray because there is a large acceleration

a) inwards which resembles a large gravitational field outwards.

b) outwards which resembles a large gravitational field outwards.

c) inwards which resembles a large gravitational field inwards.

d) outwards which resembles a large gravitational field inwards.

Explain with a freebody diagram of the water in the earth frame and in the cup’s frame.

The normal force keeps the water in circular motion. A fictitious force pulls the water into cup.

The diagrams above are not correct. The trays can’t be perfectly forizontal because a component of the normal force is needed to cancel gravity. If you swing the tray really fast – this becomes negligible.

1. Watch Alice and Bob: <http://www.perimeterinstitute.ca/en/Outreach/Alice_and_Bob_in_Wonderland/Alice_and_Bob_in_Wonderland/>
2. Why doesn’t the moon fall down?

This is a hard question to answer in words. This animation does the job beautifully. Orbital motion is another kind of freefall.

b) What keeps us stuck to the Earth?

This reinforces the idea that gravity is like an accelerating frame.

1. The astronauts on the International Space Station feel weightless because

a) the gravitational field is weaker b) the gravitational field is zero

c) the station is in freefall d) the station is moving really fast

Explain:

Even after all the previous questions, many students get this wrong. Misconceptions are hard to remove. They need to be challenged many times. Each time will have greater effect if the students have lots of time to discuss and defend their answers and to write their reasons. Answer a) can be dismissed once the students know that the orbit of the ISS is not much larger than the radius of the Earth. It is 400 km above the Earth. The radius of the Earth is 6328 km. Answer b) can be dismissed by asking what force is keeping it in orbit. Answer d) might be removed by having them consider the forces on the swinging tray again. The station and the people within are in freefall.

1. Einstein’s first breakthrough in developing general relativity came from what he described as his ‘happiest thought’ – the **equivalence principle**. This states that a gravitational field is equivalent to an accelerating frame of reference. Give examples of this.

Fig. 2: A rocket moving with constant acceleration.

Fig. 1: A rocket moving with constant velocity.

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1. The diagrams above show the position of a rocket out in space every 100 ns, going from left to right. Bob is at the rear of the rocket and sends pulses of light towards Alice in the nose of the rocket every 100 ns. Consider these different situations; Figure 1 (constant velocity), Figure 2 (acceleration) and a finally, the rocket is sitting on the surface of the Earth.

How often will Alice receive the pulses in each case and which two situations are similar?

a) every 100 ns b) more frequently than every 100 ns c) less frequently than every 100 ns

It helps to consider the three situations together. Figure 1 (constant velocity) and Figure 2 (acceleration) are clearly different, so their answers should be different. According to the equivalence principle, Figure 2 (acceleration) should be equivalent to a gravitational field and so they should have the same answer.

According to special relativity – the physics in a frame moving at constant velocity is the same as for a frame at rest. Alice should receive the pulses in the first situation every 100 ns.

In the second situation, light continues to move at c in the rocket frame. However, Alice is moving away faster and faster from that light and so the light will have to cover a greater distance. This means it takes more time. This is just the Doppler Effect. Alice will find that the pulses are delayed. You can see this by comparing pulse number five in each picture. The pulses will appear less frequently than 100 ns.

Sitting in a gravitational field is equivalent to being in a frame accelerating upward. That means that gravity slows time. If Alice sent pulses to Bob, he would receive them more frequently.