Why Start With Momentum?

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Does the curriculum start where kids naturally begin?

Kids have a natural repertoire of reasoning patterns, which are deeply meaningful to them. Some researchers identify a small number of natural schematic patterns, applicable to a wide variety of everyday events. Some concepts, such as momentum and impulse, can be supported by these natural reasoning patterns. Other concepts, such as acceleration, are not adequately dealt with by those natural reasoning patterns. A great many of the "misconceptions" described in the literature can be attributed to the mismatch between these natural reasoning patterns and accepted scientific reasoning.

Kids have a natural sense of causality, and tend to link cause effect together into meaningful wholes. Given a soccer ball, for example, a child can quite naturally predict the effect of a kick upon the ball. Even if they have a mistaken idea of how to kick the ball to get a desired result, they will learn what works relatively quickly. The inherited physics curriculum, on the other hand, decomposes everything into tiny independent pieces. In order to "understand" the motion of the ball, the student must use vector analysis, position time measurements, repeated subtractions and divisions, and so on. Not very intuitive, and often at odds with natural reasoning schemata.

Our bodies do not sense energy directly, nor do we experience gravity directly. Our bodies tend to experience impulses more accurately than other physical quantities such as force, temperature, mass, energy and so on. Uniform acceleration is not something that we can feel at all! (Ask an astronaut!) The inherited physics curriculum treats non-sensible concepts first, and leaves what is perhaps the most directly experienced quantity until the last few weeks of high school. Kids in college-bound courses get almost no opportunity to learn momentum at all.

In summary:. Momentum and impulse are more accessible to students' direct experience, more strongly related to students' sense of causality, and more easily grasped and manipulated by students' natural, schematic reasoning patterns. I believe that momentum is a kind of "gestalt" of the three main dimensions of physics. Mass, displacement and time are traditionally treated as separate entities. Human beings never experience these things in isolation, however. We sense the mass of an object by "hefting" it, moving it through space and time, and sensing how it behaves. This "gestalt" has a meaning that is greater than the sum of its isolated parts. This gestalt is, I believe, the experience of impulse and momentum.

There are two major problems with the current curriculum, vis a vis students cognition. First, the current curriculum does not correspond with everyday events. Even simple events such as kicking a ball or throwing a stone are beyond the representational power of the existing curriculum. Many university physics students lack the ability to describe everyday events in terms of sound understanding of physics. That's sound understanding, not advanced understanding. The lack of correspondence with everyday events means that students cannot easily check their school learning against the gold standard of experience. The result: experience wins, and kids forget most of their school physics within a few weeks of leaving school. Second, the curriculum requires that students learn mathematical / conceptual operations that are inherently difficult. For example, when we start with position and move to acceleration, we must attempt to teach 14 year old kids how to do a second differential of a position time function. In this task, we cannot expect to have help from the kids' natural ways of thinking.

Does the curriculum correspond with science today?

Science is not the study of nature. Science is the study of human representations of nature. We teach physics as if we were studying, for example, motion itself or electricity itself. In fact, our understanding, and our teaching, returns every time to a specific algebraic or pictorial representation of the things we are studying. Perhaps at no time since Newton have scientists been so conscious of the representational nature of science.

The most successful theories in science today are quantum theories, and relativistic theories. When the current physics curriculum was written in the '60's, these theories were still in the "egghead" or "weird science" categories. In 2004, quantum theories and relativity are foundational ideas, widely dispersed in our culture. Quantum mechanics can be most economically approached from the concept of momentum. The electron wave is widely represented as a momentum wave, and the photon is considered to be the carrier of momentum in electromagnetic interactions. Many technologies today routinely use quantum level engineering. Relativity also is more accessible from momentum platform than from a kinematics platform.

Finally, computing has changed everything. Calculus has a place, but it is no longer the central computational system. Estimation, re-iterative computing, large scale modeling etc. are more commonly used, and more commonly accessible to students, than in the past. Should we not teach concepts that make use of these methods?

In consequence of these three points, I will argue that a new physics curriculum should do the following things:

1. Students should be provided the most powerful and economical set of representations of physics phenomena. I propose that the most appropriate representation in the study of motion is the velocity time graph. This representation can be used, even by beginning students, as a means of analysis and synthesis that is far more powerful than algebraic representations.

   The second representation is the
   "*final state = initial state + change of state*"
   structure of vector representation. Not all vector quantities follow this representation. I believe that we should initially only use vector quantities which do conform to this structure in our instruction to students.

   All other representations can be postponed to later instruction. There are two instructional advantages to this approach.

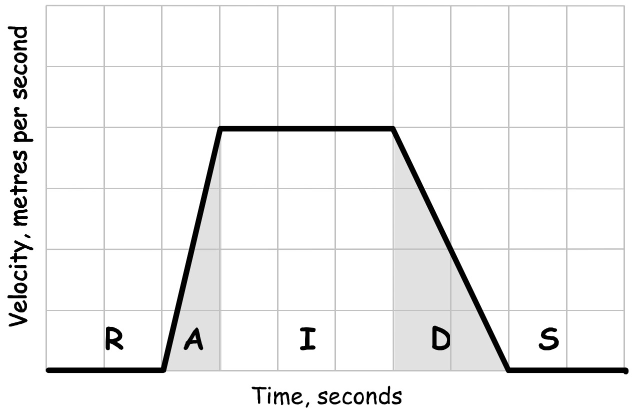
2. The curriculum should begin with momentum at the earliest possible time, and the concept of impulse following as quickly as possible. Most of the other major concepts in the study of motion can be constructed from these two concepts. force, work, kinetic energy, potential energy, acceleration, and the motion of objects in simple fields can all be derived from these two concepts.

3. Students should be taught to use those concepts and representations in a flexible way. When students construct v:t graphs of a motion, for example, they should go through a process much like the writing process in literacy courses. In response to a "physics situation", a student should construct a rough draft representation, followed by numerous refinements, until the representation provides a coherent account of the situation. Re-iterative computational methods are quite powerful in this context, and amenable to simple computer model building

This process is far more likely to result in student understanding of the tentative nature of a "scientific representation of nature" than the current model. I would expect that this process will also result in a more grounded understanding of the fundamentals of motion and action in physics.

Curriculum and Choices

Let's build the physics course, starting from the concept of momentum, and the RAIDS representation of motion.



Let us describe the motion of the ball in 5 stages.

R Rest. The ball is stationary. $v = 0$

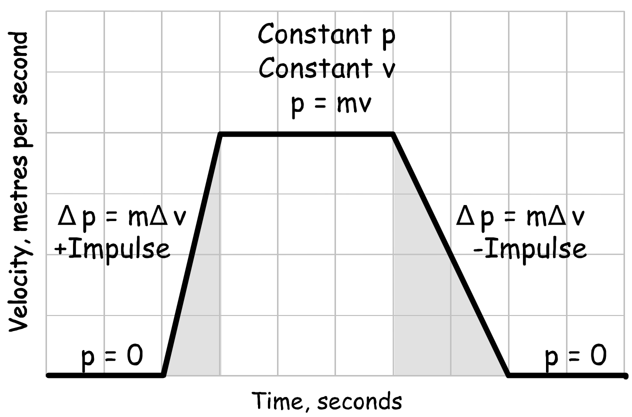
A Acceleration. The ball is speeded up.

I Inertial. The ball moves at nearly constant v

D Deceleration. The ball slows down.

S Stop. The ball comes to rest again.

Not every motion follows this simple schema, but many common situations do.



Each of the 5 stages has its own set of momentum characteristics

R $v = 0$, $p = 0$, and $\Delta p = 0$.

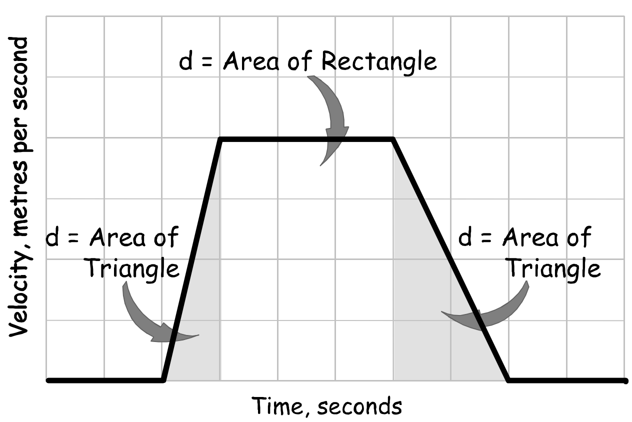
A a + impulse. Δp or $m\Delta v$.

I no impulse, no change in v or p

D an opposing – impulse. Δp or $m\Delta v$.

S $v = 0$, $p = 0$, and $\Delta p = 0$.

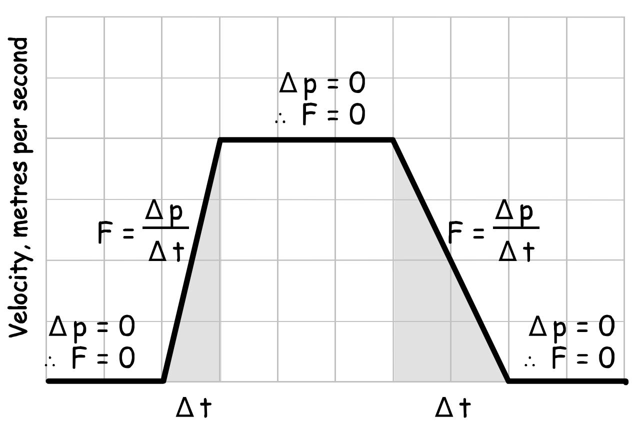
This corresponds very closely with students' intuitive understanding of motion.



The displacement of a moving object during any time interval is equivalent to the area under the v:t graph during that time interval.

This concept can replace a great deal of algebra, and supports later acquisition calculus concepts.

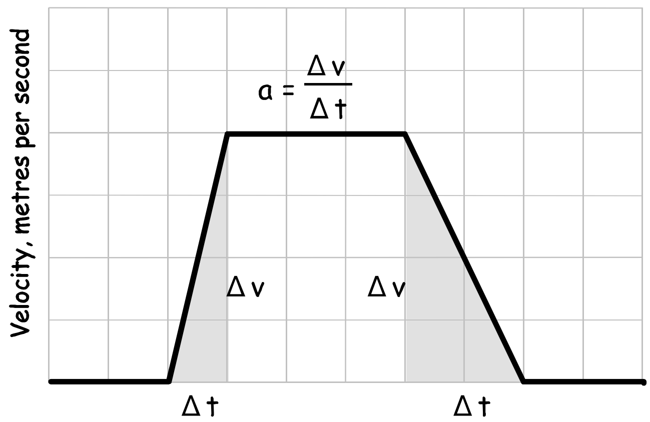
There is a wealth of detail here which is simply not available to students in conventional position time graphs, or in their algebraic representations.



Force can be understood as the "rate of change of momentum." Forces develop when things exchange momentum over time.

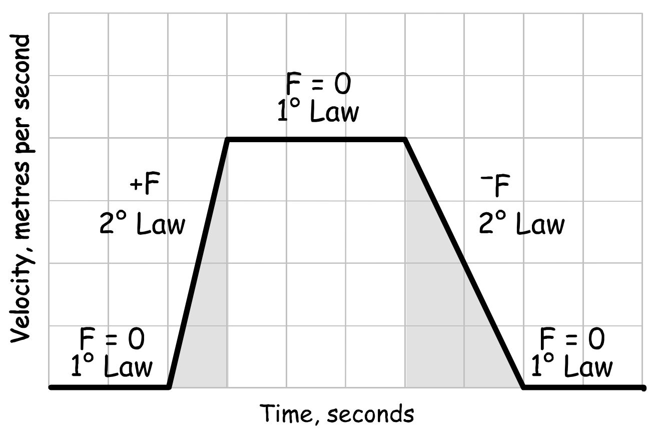
In this case, F is the total force, the net force, the unbalanced force, the sum of all the little forces, the force with no name. Any constituent forces will have a subscript to indicate their origin.

By the way, students find it easy to see where the symbol Δ is relevant by observing the triangles...



Acceleration is the rate at which velocity changes. We can define it, just the same as we always have, as the "slope of a v:t graph"

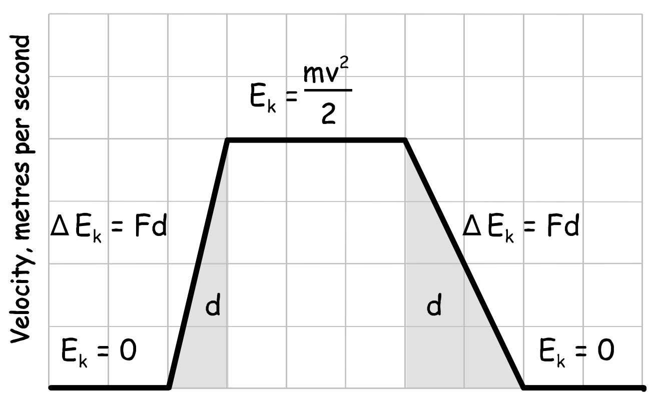
In this representation, acceleration loses its status as a fundamental quantity. In many cases, students will not need to know acceleration in order to calculate initial or final states of a system.



Newton's laws are distributed over this schema in a very simple way.

Newton's first law applies when momentum is constant. We understand that all external forces upon the ball are balanced.

Newton's second law applies when momentum is changing. Unbalanced forces provide the impulse, or change in momentum.



Work is frequently defined as "force × distance" We must then define whether we are going to deal with constituent forces or the total force, and whether each force does work or not.

Another definition, more amenable to the notion of quantum states, is that kinetic energy is a state function. E_k depends upon the state of the object, and work accomplishes the transition between states. Work is:

$$\Delta E = Fd$$

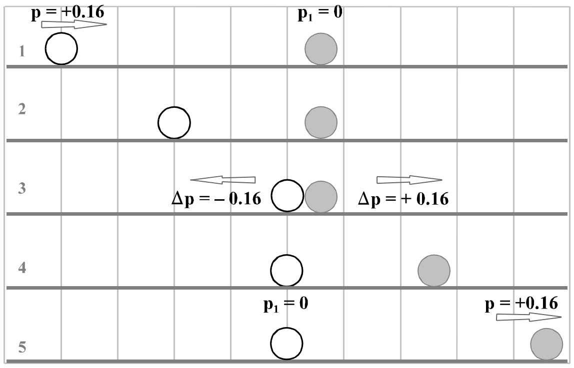
By starting with momentum, and using the RAIDS schematic representation of motion, students can organize pretty much all of linear classical mechanics. What about the conservation laws? At this time, conservation of momentum is left until the senior year of physics, grade 12. Yet the "conservation of momentum" has three extremely important characteristics for physics teachers.

First: momentum is an intuitively meaningful concept to most human beings. We experience the "oomph" of a mass in motion passing right through us into other people on the football field, the basketball court, etc. Momentum and impulse are felt in our very bones and sinews.

Second: The "conservation of momentum" is a group of concepts that have a deeper meaning together than they do apart. It is easier for a student to learn a meaningful, coherent system of concepts than to learn a large number of disconnected concepts

Third: the "conservation of momentum" is easier for students to represent, and easier for students to manipulate, than other conceptual frameworks. However, it does take exposure, and the few days that are given to momentum in the current course are not enough.

Newton's third law is usually taught as a fundamental *cause* of events, rather than as a *consequence* of even more fundamental events. If we focus upon the passage of momentum from one object to another, the impulse and momentum of the objects follows another relatively coherent pattern. We are going to modify the graphs to include only the portion of the motion that we want to explore.

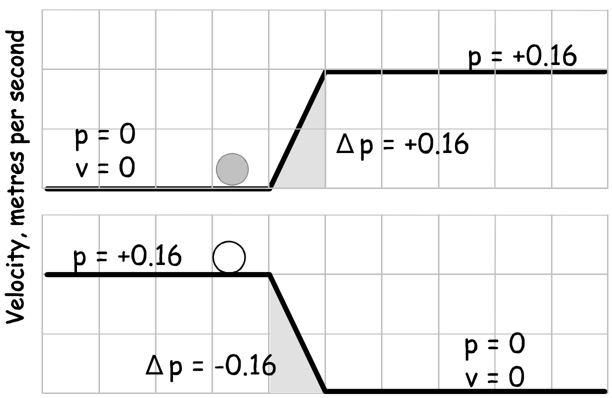


Here is a linear collision of two pool balls of equal mass. The two features of this simple collision are

1. The total momentum is the same at each instant
2. The impulse that stops the white ball is equal to and opposite to the impulse that starts the red ball.

This schematic representation is simpler, broader, and more robust than the traditional expression of Newton's 3rd Law.

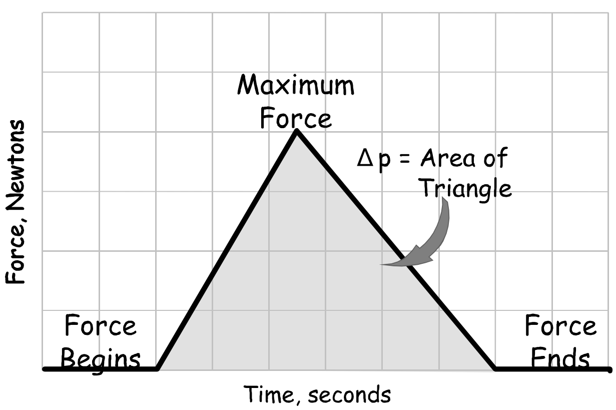
A clear understanding of the 3rd law requires previous schooling in the construction of free body diagrams, etc.



The two velocity time graphs are related. The impulse on each is equal, opposite, and of equal time duration.

Since $$\Delta p = m\Delta v = F\Delta t$$

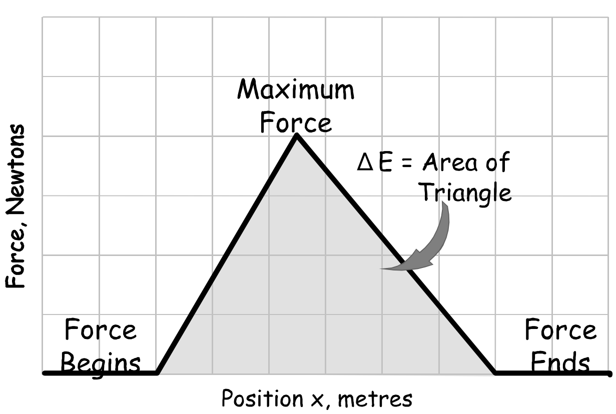
we have a very direct method of determining that the Newton's 3rd law is a consequence of conservation of momentum. Students can always compute the relative changes in velocity. With a little additional instruction, it is easy for students to compute the actual final velocities.



What if forces are changing, as the actually do in most situations? If the students are familiar with the v:t graph, then a force that changes with time is an accessible concept.

The area under the F:t graph is equivalent to impulse.

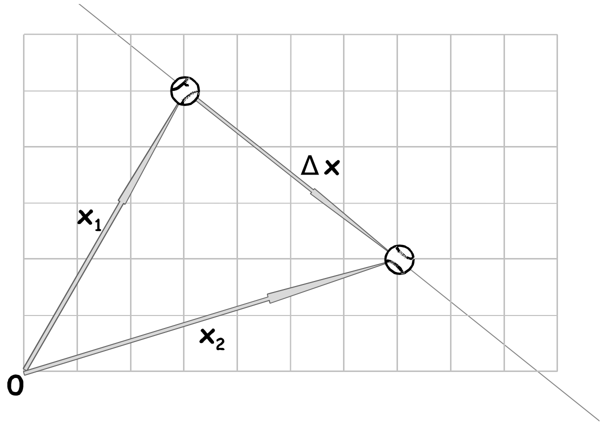
This economical representation once again gives the student enormous analytical and computational power



A similar power can be economically provided to students by using a force vs position graph. In fact, most forces can be more readily related to position (e.g. a spring) than they can be to time.

The area under the F:x graph is work, or ΔE

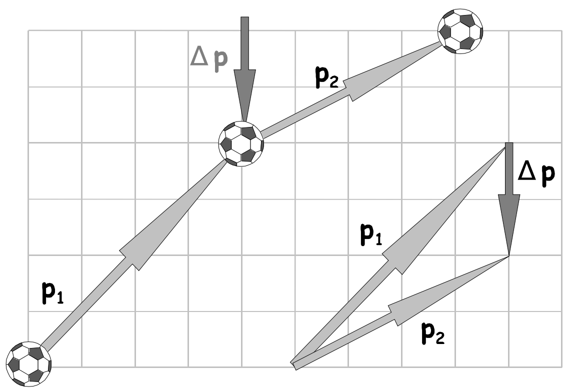
A great deal of the senior physics curriculum is devoted to 2-dimensional motion, especially free-fall trajectories in a local gravitational field, and planetary motion in a universal gravitation field. Can we deal with those issues using momentum and impulse?



Here is the fundamental structure of the vector quantity *position x* of a moving baseball. This conceptual structure is more meaningful as a *whole* than as a *set of parts*. A student can think:

> "the change from x_1 to x_2 is Δx"
> "$\Delta x = x_2 - x_1$"
> "initial plus change equals final"
> "$x_2 = x_1 + \Delta x$"

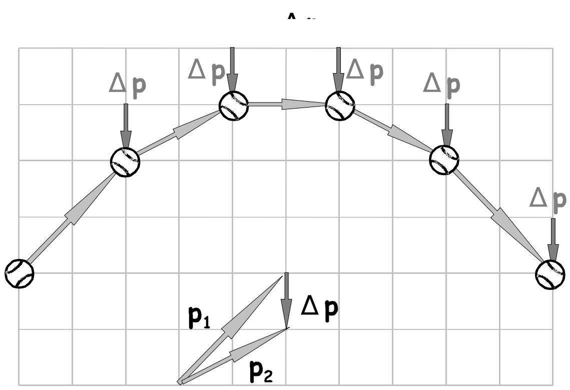
The student who can "see" all of this as a whole is in a much better position to make progress.



Here is the same structure for momentum and impulse. Students quickly grasp:

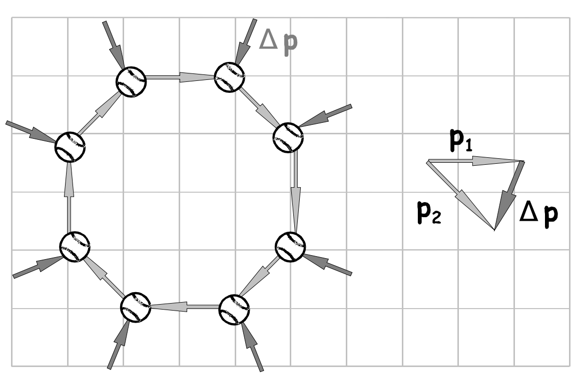
> If the ball has momentum $\mathbf{p_1}$
> and you give the ball a kick $\mathbf{\Delta p}$
> then the ball has a new momentum $\mathbf{p_2}$

The same is not intuitively true for other vector systems,



When this schema is applied repeatedly, students can quickly generate curves such as the trajectory of the baseball under repeated equal impulses of gravity.

Student reasoning strategies tend to favour situations in which the student performs some "action in the mind." Students can use this reasoning strategy to great success with this method.



In this example, the circular trajectory can be obtained in the lab by students kicking a tangentially moving ball toward the centre. Doubling the tangential speed requires that the impulse vectors also double. So

> impulses that are twice as large
> twice as many impulses per unit time
> four times the force to confine it to a circle.

Teacher Power and Student Dependency...
The representations we use have consequences.

Human understanding is a very personal thing. We may insist that the science community share representations in a coherent way, but only the individual mind can understand them.

We teachers have two very important goals in our instruction. On the one hand, we are bringing the next generation of young people into the science community's understanding of the universe. We could describe this as passing on a valuable cultural inheritance. On the other hand, we are forming the hearts and minds of this same generation. We want each student to confidently live with and contribute to their neighbours.

While many of us wish to attend to both goals, we find that the current curriculum frequently forces us to choose one goal before the other. High school physics is seen not so much as a means of personally understanding the world around us, as it is an apprenticeship into a career. I believe that we can do both, that we are not compelled to sacrifice understanding for a ticket to ride.

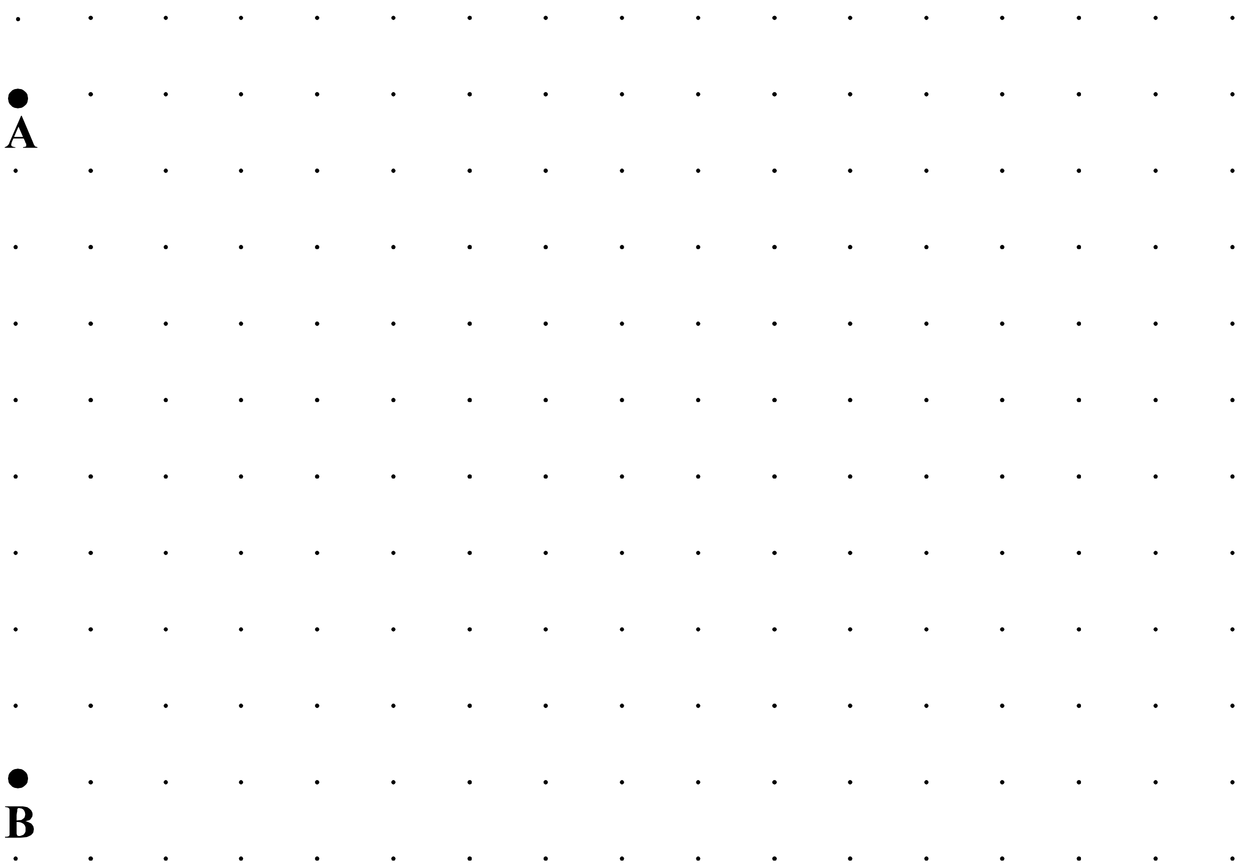
"Teaching for understanding" requires that we choose the representations that we use in our instruction very carefully. These are the advantages of starting with momentum and the RAIDS representation.

Starting with position + algebra	Starting with momentum + RAIDS
• largely analytical: students get a fixed set of data, and break the problem into bits	• largely synthetic: students develop the problems as they go.
• requires extremely accurate measurements, and very fine computation.	• students make estimates, confirm with simple measurements, and refine with calculations
• requires use of very expensive equipment, which students are unlikely to have at home	• requires use of only the simplest equipment, familiar measurements
• physics is a problem	• physics is a situation.
• right answer is outside student's control. The teacher or the textbook have the "right answer" to the problem. Students try to reproduce it.	• a coherent analysis of a situation is within the student's control. Students working on similar situations often find good agreement.
• Students do not usually have access to the assumptions made by the question constructor in order to construct a "successful" question.	• Students make their own assumptions about each situation, and they understand those assumptions.
• Only a very narrow range of problems are amenable to this method. Recall your own difficulties in constructing problems.	• An extremely broad range of problems is available for study. Physics is everywhere!
• Makes student dependent upon teacher for both problems and answers	• A robust, portable set of intellectual tools encourages student independence.

Example Vector Exercise 1: The Continuous Impulse of Gravity

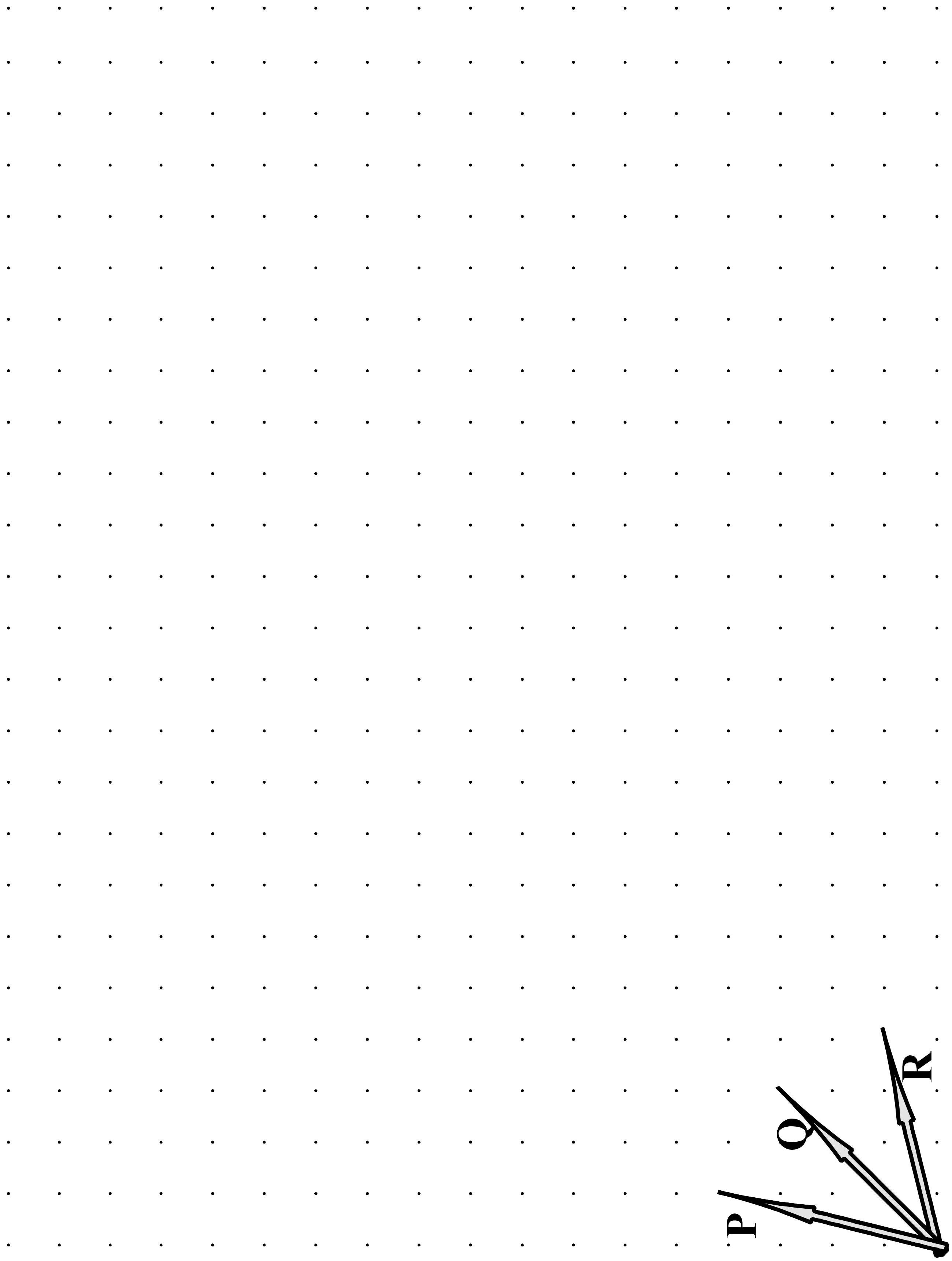
Gravity exerts a force which acts over time; thus, gravity provides a continuous impulse in a downward direction upon a body in free fall. Let us assume that the gravitational impulse is ten units of impulse per second. In one second, the **FΔt** of gravity provides exactly 10 m/s change if velocity in **mΔv** . Let us further assume that the gravitational impulse is delivered in single packets, at the end of each second.

1. A 1 kg ball, A, has instantaneous velocity [$\mathbf{v}_x = 20\ \text{ms}^{-1}$, $\mathbf{v}_y = 10\ \text{ms}^{-1}$]. Let the ball move for one second on the 10 m grid below. It. At the end of the second, deliver the gravitational impulse, calculate the new velocities, and let the ball move for another second. Repeat until the ball leaves the grid. What is the trajectory of the ball?
2. A second ball, B, is moving at [$\mathbf{v}_x = 20$, $\mathbf{v}_y = 40$]. Trace its trajectory in the same way.



Starting from the lower left corner on the 10 m grid on the following page, trace each of these trajectories.

3. Three 1 kg balls are fired at once, from the same place, with approximately the same speed, but different directions: **P** [$\mathbf{v}_x = 10$, $\mathbf{v}_y = 40$]; **Q** [$\mathbf{v}_x = 30$, $\mathbf{v}_y = 30$]; and **R** [$\mathbf{v}_x = 40$, $\mathbf{v}_y = 10$].
   a. Find the initial velocity of each, including direction. Which one will travel farthest? Trace their trajectories.
   b. Shoot one last ball, S [$\mathbf{v}_x = 20$, $\mathbf{v}_y = 50$] from the same place. Trace its trajectory.
   c. What factors govern the distance traveled by a ball in free fall?


P
Q
R

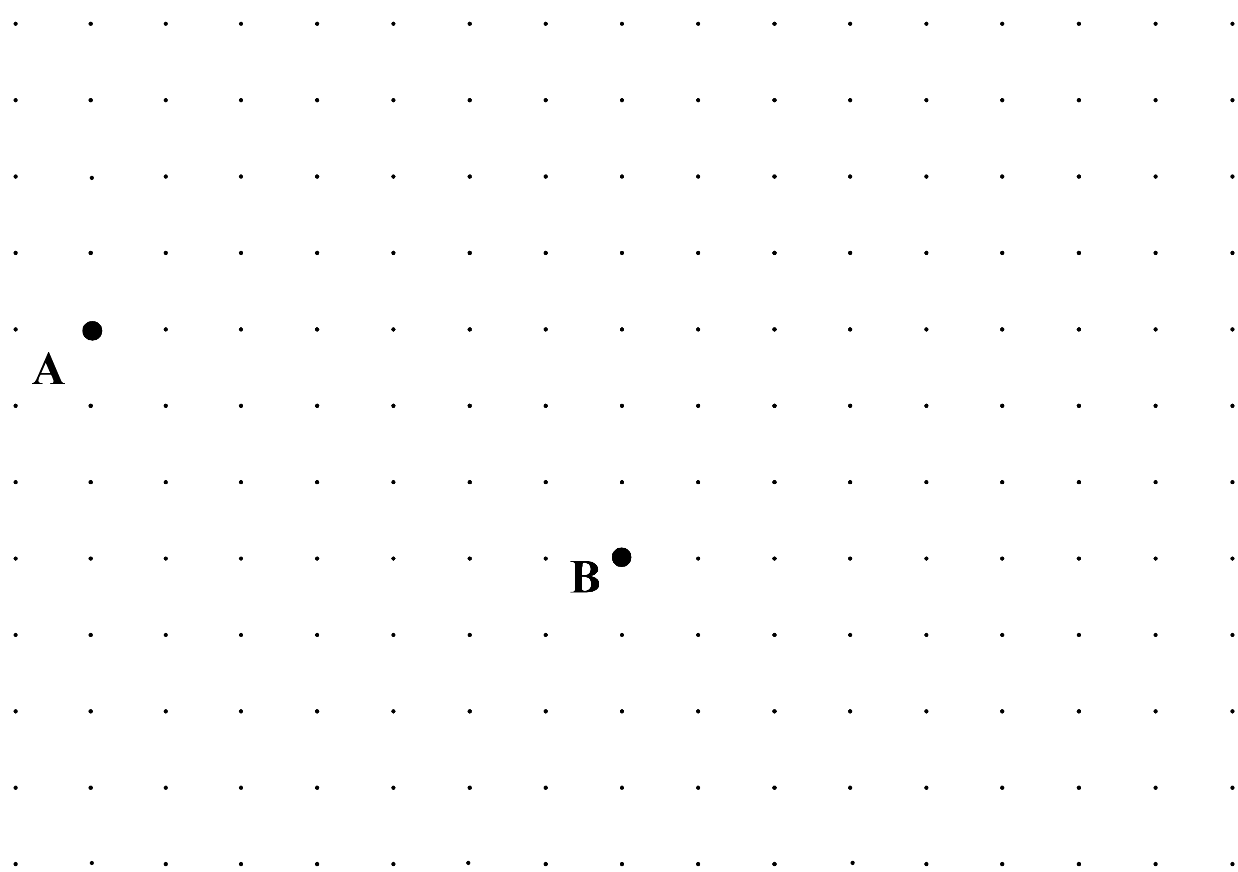
Example Vector Exercise 1: Kicking a ball around the block

1. Consider a 1 kg ball, at Point A on the 1 m grid below, initially traveling at [$\mathbf{v}_x = +2$, $\mathbf{v}_y = +2$].
   Let the ball move for two seconds, and then apply two impulses of **P** 2 N·s [S]; **Q** 2 N·s [S]
   Let the ball move for two seconds, then apply **R** 2 N·s [W] and **S** 2 N·s [W]
   Let the ball travel for two seconds, and apply **T** 2 N·s [N] and **U** 2 N·s [N]
   Let the ball move for two seconds, and apply **V** 2 N·s [E];and **W** 1 N·s [E].
   What will happen if you repeat the whole cycle from the top?

2. Start a second block at B, with initial velocity [$\mathbf{v}_x = 0$, $\mathbf{v}_y = +2$]. One second later, and at one second intervals thereafter, apply the series of impulses shown:

1,2,3	**P** = 1 N·s [E]	**P+Q** [1 N·s [E] + 1 N·s [S]]	**Q** [1 N·s [S]
4,5,6	**R** = 1 N·s [S]	**R+S** [1 N·s [S] + 1 N·s [W]]	**S** [1 N·s [W]
7,8,9	**T** = 1 N·s [W]	**T+U** [1 N·s [W] + 1 N·s [N]];	**U** [1 N·s [N]
10,11,12	**V** = 1 N·s [N]	**V+W** [1 N·s [N] + 1 N·s [E]]	**W** [1 N·s [E]

   What will happen if you repeat the whole cycle from the top?



3. If you doubled the initial velocities of the blocks, how would you have change the impulses in order to keep the blocks traveling in the same path?